4.4 A Way to Teach: Determining Nature of Rational Numbers

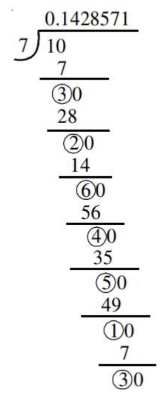
CONCEPT

Long division method can be used to determine the nature of a rational number(repeating or terminating) but isn't that method long and tedious? Can we figure out some method which is shorter and less tedious? We can dialogue with students using such questions in order to generate curiosity for the new concept

Then we can tell that they have already been using this method. It's nothing but the Prime Factorization!

Yes, we can tell whether a rational is terminating or not just by doing the prime factorization of its denominator. To explain this concept we can take the examples given in the concept post and in the video given below or any other similar example.

video duration: 6 mins



We are now ready to move on to the rational numbers whose decimal expansions are non-terminating and recurring. Once again, let us look at an example to see what is going on. We refer to Example 5, Chapter 1, from your Class IX textbook, namely, $\frac{1}{7}$. Here, remainders are 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, . . . and divisor is 7.

Notice that the denominator here, i.e., 7 is clearly not of the form 2^n5^m. Therefore, from Theorems 1.5 and 1.6, we know that $\frac{1}{7}$ will not have a terminating decimal expansion. Hence, 0 will not show up as a remainder (Why?), and the remainders will start repeating after a certain stage. So, we will have a block of digits, namely, 142857, repeating in the quotient of $\frac{1}{7}$.

What we have seen, in the case of $\frac{1}{7}$, is true for any rational number not covered by Theorems 1.5 and 1.6. For such numbers we have :

Theorem 1.7 : *Let $x = \frac{p}{q}$, where p and q are coprimes, be a rational number, such that the prime factorisation of q is not of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).*

From the discussion above, we can conclude that *the decimal expansion of every rational number is either terminating or non-terminating repeating.*

EXERCISE 1.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Download all NCERT books PDF from www.ncert.online

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can

r such that $c = dq + r$, $0 \le r < d$.

Step 2 : If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.

Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

This algorithm works because HCF (c, d) = HCF (d, r) where the symbol HCF (c, d) denotes the HCF of c and d, etc.

Example 1 : Use Euclid's algorithm to find the HCF of 4052 and 12576.

Solution :

Step 1 : Since 12576 > 4052, we apply the division lemma to 12576 and 4052, to get

$$12576 = 4052 \times 3 + 420$$

Step 2 : Since the remainder $420 \neq 0$, we apply the division lemma to 4052 and 420, to get

$$4052 = 420 \times 9 + 272$$

Step 3 : We consider the new divisor 420 and the new remainder 272, and apply the division lemma to get

$$420 = 272 \times 1 + 148$$

We consider the new divisor 272 and the new remainder 148, and apply the division lemma to get

$$272 = 148 \times 1 + 124$$

We consider the new divisor 148 and the new remainder 124, and apply the division lemma to get

$$148 = 124 \times 1 + 24$$

We consider the new divisor 124 and the new remainder 24, and apply the division lemma to get

$$124 = 24 \times 5 + 4$$

We consider the new divisor 24 and the new remainder 4, and apply the division lemma to get

$$24 = 4 \times 6 + 0$$

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 4, the HCF of 12576 and 4052 is 4.

Notice that 4 = HCF (24, 4) = HCF (124, 24) = HCF (148, 124) = HCF (272, 148) = HCF (420, 272) = HCF (4052, 420) = HCF (12576, 4052).

Euclid's division algorithm is not only useful for calculating the HCF of very large numbers, but also because it is one of the earliest examples of an algorithm that a computer had been programmed to carry out.

Remarks :

1. Euclid's division lemma and algorithm are so closely interlinked that people often call former as the division algorithm also.

2. Although Euclid's Division Algorithm is stated for only positive integers, it can be extended for all integers except zero, i.e., $b \neq 0$. However, we shall not discuss this aspect here.

Also we know.

Dividend = Divisor X Quotient + Remainder